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Limits of Some Fractional Power Exponential Functions

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Abstract: **In this article, we study the limit problems of some fractional power exponential functions. Jumarie type of Riemann-Liouville (R-L) fractional derivative, fractional L'Hospital's rule, and a new multiplication of fractional analytic functions play important roles in this paper. In fact, our results are the generalization of these results in classical calculus.**

Keyword: **fractional power exponential functions, Jumarie type of R-L fractional derivative, fractional L'Hospital's rule, new multiplication, fractional analytic functions.**

I. INTRODUCTION

Fractional calculus was proposed by Leibniz and L'Hospital in a letter dated September 30, 1695. Leibniz and L'Hospital both know about ordinary calculus and put forward the problem of noninteger differential of simple functions. Leibniz concluded the discussion by asserting that one day, the noninteger differential problem will be solved for the benefit of mankind. This unsatisfactory answer has inspired the further development of Lacroix, Fourier, Abel, Riemann, Riemann, Liouville and others in the past 300 years.

In recent years, fractional calculus has become an increasingly popular research area due to its effective applications in different scientific fields such as economics, viscoelasticity, physics, dynamics, biology, control theory, and so on [1-7]. However, the definition of fractional derivative is not unique. The commonly used definitions include Riemann-Liouville (R-L) fractional derivative, Caputo fractional derivative, Grunwald-Letnikov (G-L) fractional derivative, and Jumarie's modified R-L fractional derivative [8-11]. Since Jumarie type of R-L fractional derivative helps to avoid non-zero fractional derivative of constant function, it is easier to use this definition to connect fractional calculus with traditional calculus.

This paper studies the limit problems of some fractional power exponential functions. Jumarie's modified R-L fractional derivative, fractional L'Hospital's rule, and a new multiplication of fractional analytic functions play important roles in this article. In fact, our results are the generalization of these results in ordinary calculus.

II. DEFINITIONS AND PROPERTIES

Firstly, the fractional derivative used in this paper and its properties are introduced below.

Definition 2.1 ([12])**:** Let $0 < \alpha \leq 1$, and x_0 be a real number. The Jumarie type of Riemann-Liouville (R-L) α-fractional derivative is defined by

$$\left({}_{x_0}D_x^{\alpha}\right)[f(x)] = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_{x_0}^{x}\frac{f(t)-f(x_0)}{(x-t)^{\alpha}}dt\,. \quad (1)$$

where $\Gamma(\ )$ is the gamma function.

Proposition 2.2 ([13])**:** *If* α, β, x_0, c *are real numbers and* $\beta \geq \alpha > 0$, *then*

$$\left({}_0D_x^{\alpha}\right)[x^{\beta}]=\frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}x^{\beta-\alpha}, \tag{2}$$

and

$$\left({}_0D_x^{\alpha}\right)[c]=0. \tag{3}$$

Next, we introduce the definition of fractional analytic function.

Definition 2.3 ([14])**:** Suppose that x and a_k are real numbers for all k, and $0<\alpha\leq 1$. If the function $f_{\alpha}:[a,b]\to R$ can be expressed as an α-fractional power series, that is, $f_{\alpha}(x^{\alpha})=\sum_{k=0}^{\infty}\frac{a_k}{\Gamma(k\alpha+1)}x^{k\alpha}$, then we say that $f_{\alpha}(x^{\alpha})$ is α-fractional analytic function.

In the following, we introduce a new multiplication of fractional analytic functions.

Definition 2.4 ([15])**:** If $0<\alpha\leq 1$. Suppose that $f_{\alpha}(x^{\alpha})$ and $g_{\alpha}(x^{\alpha})$ are two α-fractional analytic functions,

$$f_{\alpha}(x^{\alpha})=\sum_{k=0}^{\infty}\frac{a_k}{\Gamma(k\alpha+1)}x^{k\alpha}=\sum_{k=0}^{\infty}\frac{a_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes k}, \tag{4}$$

$$g_{\alpha}(x^{\alpha})=\sum_{k=0}^{\infty}\frac{b_k}{\Gamma(k\alpha+1)}x^{k\alpha}=\sum_{k=0}^{\infty}\frac{b_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes k}. \tag{5}$$

Then we define

$$\begin{aligned}&f_{\alpha}(x^{\alpha})\otimes g_{\alpha}(x^{\alpha})\\&=\sum_{k=0}^{\infty}\frac{a_k}{\Gamma(k\alpha+1)}x^{k\alpha}\otimes\sum_{k=0}^{\infty}\frac{b_k}{\Gamma(k\alpha+1)}x^{k\alpha}\\&=\sum_{k=0}^{\infty}\frac{1}{\Gamma(k\alpha+1)}\left(\sum_{m=0}^{k}\binom{k}{m}a_{k-m}b_m\right)x^{k\alpha}.\end{aligned} \tag{6}$$

Equivalently,

$$\begin{aligned}&f_{\alpha}(x^{\alpha})\otimes g_{\alpha}(x^{\alpha})\\&=\sum_{k=0}^{\infty}\frac{a_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes k}\otimes\sum_{k=0}^{\infty}\frac{b_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes k}\\&=\sum_{k=0}^{\infty}\frac{1}{k!}\left(\sum_{m=0}^{k}\binom{k}{m}a_{k-m}b_m\right)\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes k}.\end{aligned} \tag{7}$$

Definition 2.5 ([15])**:** Assume that $0<\alpha\leq 1$, and $f_{\alpha}(x^{\alpha})$, $g_{\alpha}(x^{\alpha})$ are α-fractional analytic functions,

$$f_{\alpha}(x^{\alpha})=\sum_{k=0}^{\infty}\frac{a_k}{\Gamma(k\alpha+1)}x^{k\alpha}=\sum_{k=0}^{\infty}\frac{a_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes k}, \tag{8}$$

$$g_{\alpha}(x^{\alpha})=\sum_{k=0}^{\infty}\frac{b_k}{\Gamma(k\alpha+1)}x^{k\alpha}=\sum_{k=0}^{\infty}\frac{b_k}{k!}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes k}. \tag{9}$$

The compositions of $f_{\alpha}(x^{\alpha})$ and $g_{\alpha}(x^{\alpha})$ are defined by

$$(f_{\alpha}\circ g_{\alpha})(x^{\alpha})=f_{\alpha}\left(g_{\alpha}(x^{\alpha})\right)=\sum_{k=0}^{\infty}\frac{a_k}{k!}\left(g_{\alpha}(x^{\alpha})\right)^{\otimes k}, \tag{10}$$

and

$$(g_{\alpha}\circ f_{\alpha})(x^{\alpha})=g_{\alpha}\left(f_{\alpha}(x^{\alpha})\right)=\sum_{k=0}^{\infty}\frac{b_k}{k!}\left(f_{\alpha}(x^{\alpha})\right)^{\otimes k}. \tag{11}$$

Definition 2.6 ([15])**:** Let $0<\alpha\leq 1$. If $f_{\alpha}(x^{\alpha})$, $g_{\alpha}(x^{\alpha})$ are two α-fractional analytic functions satisfies

$$(f_{\alpha}\circ g_{\alpha})(x^{\alpha})=(g_{\alpha}\circ f_{\alpha})(x^{\alpha})=\frac{1}{\Gamma(\alpha+1)}x^{\alpha}. \tag{12}$$

Then $f_{\alpha}(x^{\alpha})$, $g_{\alpha}(x^{\alpha})$ are called inverse functions of each other.

Some fractional analytic functions are introduced below.

Definition 2.7 ([16])**:** If $0<\alpha\leq 1$, and x, x_0 are real numbers. The α-fractional exponential function is defined by

$$E_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{x^{k\alpha}}{\Gamma(k\alpha+1)} = \sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes k}. \quad (13)$$

And the α-fractional logarithmic function $Ln_\alpha(x^\alpha)$ is the inverse function of $E_\alpha(x^\alpha)$. Moreover, the α-fractional cosine and sine function are defined respectively as follows:

$$cos_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k\alpha}}{\Gamma(2k\alpha+1)} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes 2k}, \quad (14)$$

and

$$sin_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{(2k+1)\alpha}}{\Gamma((2k+1)\alpha+1)} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes (2k+1)}. \quad (15)$$

Definition 2.8 ([17])**:** If $0 < \alpha \le 1$. If $u_\alpha(x^\alpha)$, $w_\alpha(x^\alpha)$ are two α-fractional analytic functions. Then the α-fractional power exponential function $u_\alpha(x^\alpha)^{\otimes w_\alpha(x^\alpha)}$ is defined by

$$u_\alpha(x^\alpha)^{\otimes w_\alpha(x^\alpha)} = E_\alpha\left(w_\alpha(x^\alpha) \otimes Ln_\alpha\left(u_\alpha(x^\alpha)\right)\right). \quad (16)$$

Theorem 2.9 ([18])**:** *Let* $0 < \alpha \le 1$, *then the* α*-fractional exponential function*

$$E_\alpha(x^\alpha) = e_\alpha^{\otimes \frac{1}{\Gamma(\alpha+1)}x^\alpha}, \quad (17)$$

where $e_\alpha = E_\alpha(1) = \sum_{k=0}^{\infty} \frac{[\Gamma(\alpha+1)]^k}{\Gamma(k\alpha+1)}$.

Theorem 2.10 ([19]) (fractional L'Hospital's rule)**:** *Suppose that* $0 < \alpha \le 1$, c *is a real number, and* $f_\alpha(x^\alpha)$, $g_\alpha(x^\alpha)$, $[g_\alpha(x^\alpha)]^{\otimes -1}$ *are* α*-fractional analytic functions at* $x = c$. *If* $\lim_{x\to c} f_\alpha(x^\alpha) = \lim_{x\to c} g_\alpha(x^\alpha) = 0$, *or* $\lim_{x\to c} f_\alpha(x^\alpha) = \pm\infty$, *and* $\lim_{x\to c} g_\alpha(x^\alpha) = \pm\infty$. *Assume that* $\lim_{x\to c} f_\alpha(x^\alpha) \otimes [g_\alpha(x^\alpha)]^{\otimes -1}$ *and* $\lim_{x\to c}\left({}_cD_x^\alpha\right)[f_\alpha(x^\alpha)] \otimes \left[\left({}_cD_x^\alpha\right)[g_\alpha(x^\alpha)]\right]^{\otimes -1}$ *exist,* $\left({}_cD_x^\alpha\right)[g_\alpha(x^\alpha)](c) \neq 0$. *Then*

$$\lim_{x\to c} f_\alpha(x^\alpha) \otimes [g_\alpha(x^\alpha)]^{\otimes -1} = \lim_{x\to c}\left({}_cD_x^\alpha\right)[f_\alpha(x^\alpha)] \otimes \left[\left({}_cD_x^\alpha\right)[g_\alpha(x^\alpha)]\right]^{\otimes -1}. \quad (18)$$

III. RESULTS AND EXAMPLES

The followings are main results in this paper.

Theorem 3.1: *Let* $0 < \alpha \le 1$, *then*

$$E_\alpha(x^\alpha) = \lim_{n\to\infty}\left(1 + \frac{1}{n} \cdot \frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes n}. \quad (19)$$

Proof Using fractional L'Hospital's rule yields

$$Ln_\alpha\left(\lim_{n\to\infty}\left(1 + \frac{1}{n} \cdot \frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes n}\right)$$

$$= \lim_{n\to\infty}\left[Ln_\alpha\left(\left(1 + \frac{1}{n} \cdot \frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes n}\right)\right]$$

$$= \lim_{n\to\infty}\left[n \cdot Ln_\alpha\left(1 + \frac{1}{n} \cdot \frac{1}{\Gamma(\alpha+1)}x^\alpha\right)\right]$$

$$= \frac{1}{\Gamma(\alpha+1)}x^\alpha \cdot \lim_{n\to\infty}\left[Ln_\alpha\left(1 + \frac{1}{n} \cdot \frac{1}{\Gamma(\alpha+1)}x^\alpha\right) \otimes \left(\frac{1}{n} \cdot \frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes -1}\right]$$

$$= \frac{1}{\Gamma(\alpha+1)}x^\alpha \cdot \lim_{s\to 0}\left[Ln_\alpha\left(1 + \frac{1}{\Gamma(\alpha+1)}s^\alpha\right) \otimes \left(\frac{1}{\Gamma(\alpha+1)}s^\alpha\right)^{\otimes -1}\right]$$

$$= \frac{1}{\Gamma(\alpha+1)}x^\alpha. \quad (20)$$

It follows that

$$E_\alpha\left(Ln_\alpha\left(\lim_{n\to\infty}\left(1+\frac{1}{n}\cdot\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes n}\right)\right)=E_\alpha\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)=E_\alpha(x^\alpha). \tag{21}$$

And hence,

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\cdot\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes n}=E_\alpha(x^\alpha).$$ Q.e.d.

Theorem 3.2: *Suppose that* $0<\alpha\le 1$, *then*

$$e_\alpha=\lim_{t\to 0}\left\{\left(1+\frac{1}{\Gamma(\alpha+1)}t^\alpha\right)^{\otimes\left(\frac{1}{\Gamma(\alpha+1)}t^\alpha\right)^{\otimes -1}}\right\}. \tag{22}$$

Proof By Theorem 3.1,

$$\begin{aligned}&E_\alpha(x^\alpha)\\&=\lim_{n\to\infty}\left(1+\frac{1}{n}\cdot\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes n}\\&=\lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\cdot\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes\left(\frac{1}{n}\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes -1}}\right\}^{\otimes\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)}\\&=\left\{\lim_{n\to\infty}\left(1+\frac{1}{n}\cdot\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes\left(\frac{1}{n}\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes -1}}\right\}^{\otimes\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)}\\&=\left\{\lim_{t\to 0}\left(1+\frac{1}{\Gamma(\alpha+1)}t^\alpha\right)^{\otimes\left(\frac{1}{\Gamma(\alpha+1)}t^\alpha\right)^{\otimes -1}}\right\}^{\otimes\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)}.\end{aligned} \tag{23}$$

Thus by Theorem 2.9, we have

$$e_\alpha=\lim_{t\to 0}\left(1+\frac{1}{\Gamma(\alpha+1)}t^\alpha\right)^{\otimes\left(\frac{1}{\Gamma(\alpha+1)}t^\alpha\right)^{\otimes -1}}.$$ Q.e.d.

Theorem 3.3: *If* $0<\alpha\le 1$, *then*

$$e_\alpha=\lim_{y\to\infty}\left(1+\left(\frac{1}{\Gamma(\alpha+1)}y^\alpha\right)^{\otimes -1}\right)^{\otimes\left(\frac{1}{\Gamma(\alpha+1)}y^\alpha\right)}. \tag{24}$$

Proof Let $\left(\frac{1}{\Gamma(\alpha+1)}y^\alpha\right)^{\otimes -1}=\frac{1}{\Gamma(\alpha+1)}t^\alpha$, then by Theorem 3.2,

$$\begin{aligned}e_\alpha&=\lim_{t\to 0}\left(1+\frac{1}{\Gamma(\alpha+1)}t^\alpha\right)^{\otimes\left(\frac{1}{\Gamma(\alpha+1)}t^\alpha\right)^{\otimes -1}}\\&=\lim_{y\to\infty}\left(1+\left(\frac{1}{\Gamma(\alpha+1)}y^\alpha\right)^{\otimes -1}\right)^{\otimes\left(\frac{1}{\Gamma(\alpha+1)}y^\alpha\right)}.\end{aligned}$$ Q.e.d.

Next, we provide some examples to illustrate how to use the above results to solve the limit problems of fractional power exponential functions.

Example 3.4: Let $0<\alpha\le 1$ and $(-1)^\alpha=-1$. Find the limit

$$\lim_{y\to\infty}\left(1-\left(\frac{1}{\Gamma(\alpha+1)}y^\alpha\right)^{\otimes -1}\right)^{\otimes\left(\frac{1}{\Gamma(\alpha+1)}y^\alpha\right)}.$$

Solution Let $p=-y$, then $\frac{1}{\Gamma(\alpha+1)}p^\alpha=\frac{1}{\Gamma(\alpha+1)}(-y)^\alpha=-\frac{1}{\Gamma(\alpha+1)}y^\alpha$, then by Theorem 3.3,

$$\lim_{y\to\infty}\left(1-\left(\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right)^{\otimes -1}\right)^{\otimes\left(\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right)}$$

$$=\lim_{y\to\infty}\left(1+\left(-\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right)^{\otimes -1}\right)^{\otimes\left(\frac{1}{\Gamma(\alpha+1)}y^{\alpha}\right)}$$

$$=\lim_{p\to-\infty}\left(1+\left(\frac{1}{\Gamma(\alpha+1)}p^{\alpha}\right)^{\otimes -1}\right)^{\otimes -\left(\frac{1}{\Gamma(\alpha+1)}p^{\alpha}\right)}$$

$$=\lim_{p\to-\infty}\left\{\left[\left(1+\left(\frac{1}{\Gamma(\alpha+1)}p^{\alpha}\right)^{\otimes -1}\right)^{\otimes\left(\frac{1}{\Gamma(\alpha+1)}p^{\alpha}\right)}\right]^{\otimes -1}\right\}$$

$$=\frac{1}{e_{\alpha}}. \tag{25}$$

Example 3.5: Assume that $0<\alpha\le 1$. Evaluate the limit

$$\lim_{x\to 0}\left[1+2\cdot\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 3\cdot\left(sin_{\alpha}(x^{\alpha})\right)^{\otimes -1}}.$$

Solution By fractional L'Hospital's rule, we have

$$\lim_{x\to 0}\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\otimes\left(sin_{\alpha}(x^{\alpha})\right)^{\otimes -1}=1. \tag{26}$$

Therefore, by Theorem 3.2,

$$\lim_{x\to 0}\left[1+2\cdot\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes 3\cdot\left(sin_{\alpha}(x^{\alpha})\right)^{\otimes -1}}$$

$$=\lim_{x\to 0}\left[1+2\cdot\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right]^{\otimes\left(2\cdot\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes -1}6\cdot\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\otimes\left(sin_{\alpha}(x^{\alpha})\right)^{\otimes -1}}$$

$$=e_{\alpha}^{\;6\cdot\lim_{x\to 0}\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\otimes\left(sin_{\alpha}(x^{\alpha})\right)^{\otimes -1}}$$

$$=e_{\alpha}^{\;6}. \tag{27}$$

Example 3.6: Let $0<\alpha\le 1$. Find the limit

$$\lim_{x\to\infty}\left[\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+3\right)\otimes\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+6\right)^{\otimes -1}\right]^{\otimes\frac{1}{2}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}-1\right)}.$$

Solution By Theorem 3.3,

$$\lim_{x\to\infty}\left[\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+3\right)\otimes\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+6\right)^{\otimes -1}\right]^{\otimes\frac{1}{2}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}-1\right)}$$

$$=\lim_{x\to\infty}\left[1+(-3)\cdot\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+6\right)^{\otimes -1}\right]^{\otimes\frac{1}{2}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}-1\right)}$$

$$=\lim_{x\to\infty}\left[1+(-3)\cdot\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+6\right)^{\otimes -1}\right]^{\otimes\frac{1}{-3}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+6\right)\otimes\frac{-3}{2}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}-1\right)\otimes\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+6\right)^{\otimes -1}}$$

$$=\lim_{x\to\infty}\left\{\left[1+(-3)\cdot\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+6\right)^{\otimes -1}\right]^{\otimes\frac{1}{-3}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+6\right)}\right\}^{\otimes\left[\frac{-3}{2}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}-1\right)\otimes\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}+6\right)^{\otimes -1}\right]}$$

$$= \left\{ \lim_{x \to \infty} \left[1 + (-3) \cdot \left(\frac{1}{\Gamma(\alpha+1)} x^{\alpha} + 6 \right)^{\otimes -1} \right]^{\otimes \frac{1}{-3} \left(\frac{1}{\Gamma(\alpha+1)} x^{\alpha} + 6 \right)} \right\}^{\lim_{x \to \infty} \left[\frac{-3}{2} \left(\frac{1}{\Gamma(\alpha+1)} x^{\alpha} - 1 \right) \otimes \left(\frac{1}{\Gamma(\alpha+1)} x^{\alpha} + 6 \right)^{\otimes -1} \right]}$$

$$= {e_{\alpha}}^{-\frac{3}{2}}. \tag{28}$$

IV. CONCLUSION

In this paper, we find the limits of some fractional power exponential functions. Jumarie's modified R-L fractional derivative, fractional L'Hospital's rule, and a new multiplication of fractional analytic functions play important roles in this paper. In fact, our results are the generalization of these results in traditional calculus. In the future, we will expand our research fields to fractional differential equations and engineering mathematics.

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